Simple and Cantilever K-Trusses Analyzed

Part I—Formulas Derived and Influence Lines Drawn for Chord and Web Stresses and Applied to the Determination of Maximum Stresses

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THE essential feature of the K type of truss is the use of two main diagonal web members in each panel, arranged in such a way that the stresses are statically determinate. The consequent division of the shear results in lower sectional areas and less complicated details of web members and their connections than obtain, for example, in the Pratt truss with sub-panels; while it has the advantage over other types with two main diagonals in one panel, such as the double-intersection Warren truss, of being statically determinate. In view of these and other advantages, such as comparative freedom from secondary stresses and adaptability to economical and rapid erection, it may confidently be assumed that the K-truss will command wide attention among designers, especially for long spans, to which it is peculiarly adapted. The advantages of the K-truss and the reasons for its use in the Quebec Bridge are fully explained in a paper by Ralph Modjeski on "Design of Large Bridges, with Special Reference to Quebec Bridge" in the Engineering Record of Sept. 20, 1913, page 321.

Analysis for Primary Stresses—Assumptions

In the analysis which follows, the endeavor has been not only to develop direct, concise methods of determining the so-called "primary" stresses, but to emphasize those features which differ from the analyses of the usual types of trusses given in text-books on framed structures. Influence lines for stress are constructed, since these show most clearly the relation between stress and position of load and are very useful in discussing maximum stresses. The truss is first considered to act as a simple span, and the methods developed are then applied to a cantilever arm. These methods, however, are independent of end conditions and may be applied, for example, to a continuous girder after the reactions have been determined. As usual in problems dealing with influence lines, the stresses are computed for a single concentrated load of unit intensity moving over the structure.

The truss illustrated in Fig. 1 is similar to the anchor and cantilever arms of the Quebec Bridge. Although a truss acting only as a simple span would be symmetrical about its center line, it is convenient to use the same truss for both simple span and cantilever arm. Moreover, the anchor arm acts as a simple span under its own weight and loading. The following assumptions have been made in the interest of a general treatment:

1. That the points c′, e′, g′, etc., do not lie in the same straight line (in the Quebec Bridge they are in a line parallel to the grade line);
2. That such members as c′E, d′e′ and e′G are not parallel;
3. That the panels, and their divisions into sub-panels, are unequal in length; and finally,
4. That the slopes of the chords are unequal.

The analysis will be confined to the chord and web members in the panel EG, these being typical of the entire truss. The notation required is shown in Fig. 1. The values of OO_1, OO_2, and O_1O_2 are self-evident. The solution for z, the perpendicular distance from O_1 to e′G, is given in Fig. 1 (a). The value of z_1 follows by analogy.

Top-Chord Stress EG—Imagine the truss to be divided into two parts by section 1-1, Fig. 1, cutting five members, and forces to be applied at the ends of these members equal to the stresses in them before the truss was severed. These stresses and the external forces to either side of the section form a system of co-planar forces in equilibrium, and the algebraic sum of their moments about any point in their plane is therefore equal to zero. To find stress EG, moments should be taken about e, since three of the other four stresses, Ee′, e′e and eg, traverse e and are thus eliminated from the equation of moments. The fourth stress, d′e′, is evidently zero except when there is a floorbeam concentration at d′—that is, for loads in panel CE. The determination of stress EG may therefore conveniently be divided into two parts—(1) for loads outside of panel CE, in which case EG is the only stress in the moment equation, and (2) for loads in panel CE, in which case stress d′e′ enters that equation and must therefore be determined.

The first part presents a problem similar to that of the simple Pratt truss. Thus, with the unit load in any position from e′ to the right, at a distance x from the right support, the only external force to the left of section 1-1 is the reaction, R_a at a. Since stress d′e′ is zero, the algebraic sum of the moments about e of R_a and stress EG must equal zero. It will be more convenient to solve for the horizontal component, from which the stress itself may readily be found by multiplying by P'/P. Therefore resolve stress EG into vertical and horizontal components at E; the vertical component has no moment about e; the lever arm of the horizontal component is h. Then for loads from e′ to the right,

$$R_a \times r - \text{horizontal component EG} \times h = 0,$$

or since $R_a = \dfrac{x}{l}$,

$$\text{horizontal component EG} = \frac{x}{l} \times \frac{r}{h} \qquad (1)$$

In equation 1, x may have any value from zero to $l - r$. Stress EG is compressive, opposing the moment of R_a about e.

Similarly, by placing the unit load in any position from c′ to the left, at a distance x' from the left support, and taking moments about e of the forces to the right of section 1-1, the following equation is obtained:

$$\text{horizontal component EG} = \frac{x'}{l} \times \frac{l - r}{h} \qquad (2)$$

in which x' may have any value from zero to $r - P_1$.

For the unit load in panel CE—placed for convenience at d′—the value of the tensile stress d′e′ is i''/h_1, since the triangle d′e′e may be considered as a polygon for the forces at d′, the length e′e ($= h_1$) representing the unit load and length d′e′ ($= i''$) the stress d′e′. Expressing the moment of stress d′e′ about e as its horizontal component, p_1/h_1, multiplied by h_1, the following equation of moments of stresses and external forces to the left of section 1-1 about e may be written:

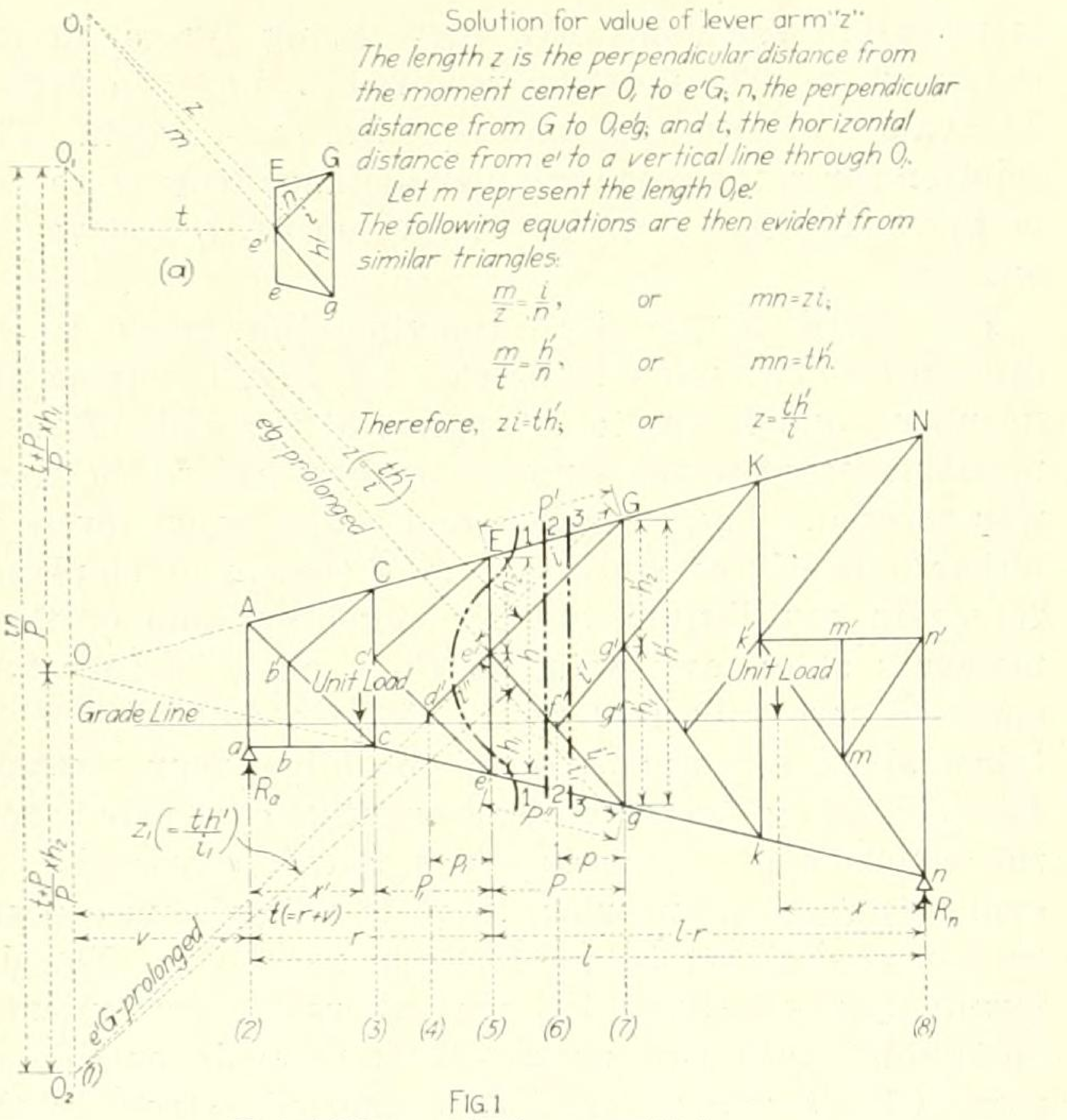

FIG. 1

Typical K-Truss, with Necessary Notation

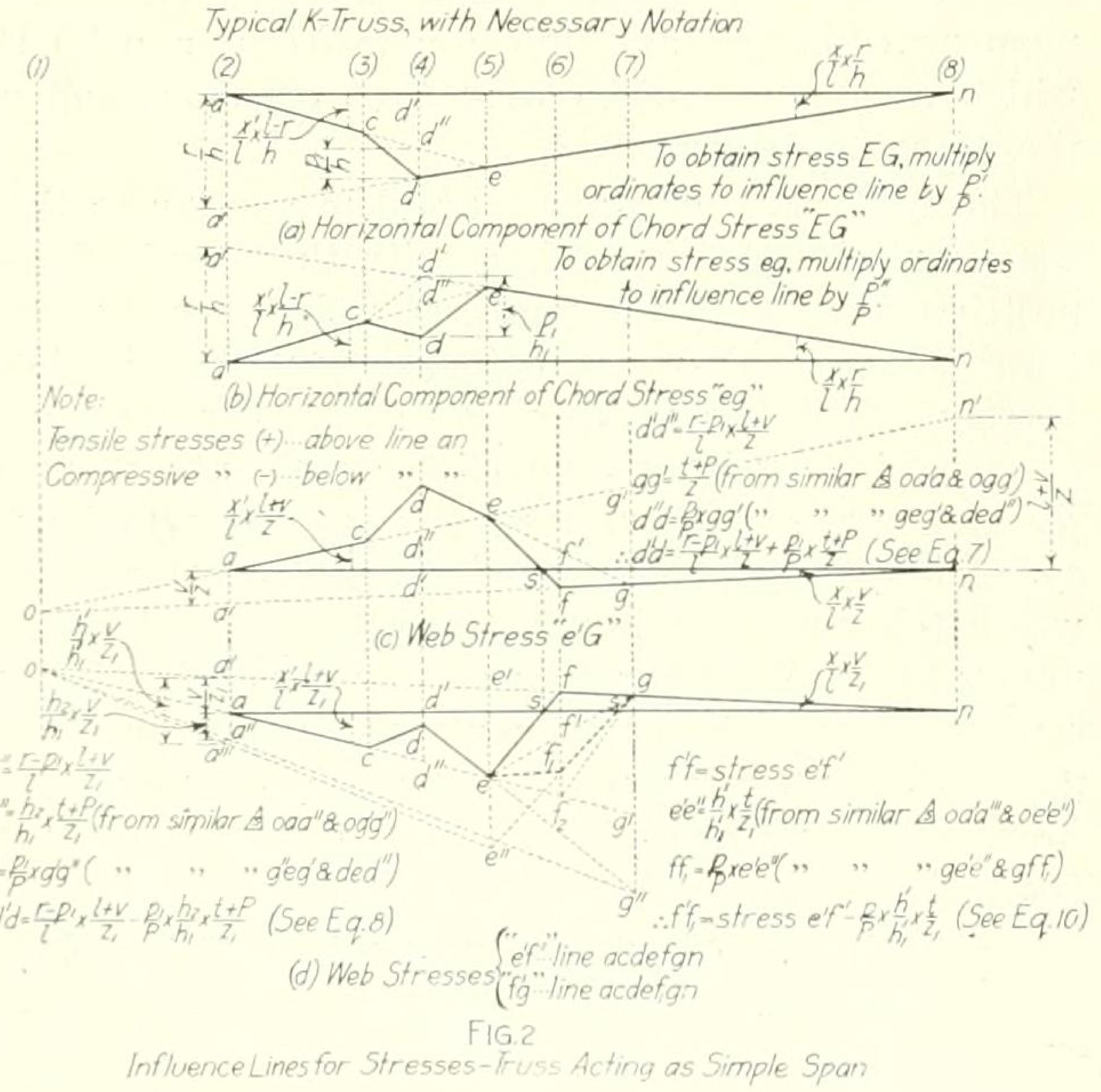

FIG. 2

Influence Lines for Stresses—Truss Acting as Simple Span

$$R_a \times r - 1 \times p_1 + \frac{p_1}{h_1} \times h_1 -$$

$$\text{horizontal component EG} \times h = 0,$$

or since $R_a = \dfrac{l - r + p_1}{l}$,

$$\text{horizontal component EG} = \frac{l - r + p_1}{l} \times \frac{r}{h} \qquad (3)$$

Equations 1, 2 and 3 may now be used to construct the influence line for the horizontal component of stress EG. Referring to Fig. 2 (a), ne and ac are graphs of equations 1 and 2 respectively. These lines may most conveniently be drawn by observing, first, that the intercept aa′ on line (2) of ne prolonged is r/h, the value of equation 1 for $x = l$; and second, that equations 1 and 2 are identical for values of $x = l - r$ and $x' = r$ respectively, so that ac prolonged traverses e. Next, lay off on line (4) the ordinate d′d equal to equation 3; then, since an influence line for any function is a straight line between successive panel points in the floor system, the lines cd and de complete the influence line across panel CE. But for $x = l - r + p_1$, equation 1 becomes identical with equation 3; therefore d must fall on ne prolonged. This is also evident from the fact that for any position of the unit load between e′ and d′ the moments about e of the load and stress d′e′ neutralize each other, thus leaving in the moment equation only R_a and stress EG, as in equation 1.

INFLUENCE LINES FOR CHORD STRESS EG

The complete influence line is acdn. If d′e′ were removed, the influence line would be acen, which is typical of the simple Pratt truss. Incidentally, this is the form of the influence line for stresses CE and ce in Fig. 1, since a section similar to 1-1 through these members traverses only four members—there being no member at c′ corresponding to d′e′. An ordinate between ce and cde, Fig. 2 (a), therefore represents an increase in the horizontal component of the chord stress EG induced by the secondary system in panel CE, the value of dd″ being equal to p_1/h. An analysis of stress EG of the sub-Pratt (Pettit) truss illustrated in Fig. 3 (a) leads to an influence line identical in form with acdn, the only difference being that here the secondary system which causes the increase in stress referred to above is that in panel EG, so that the triangle corresponding to cde, Fig. 2 (a), will appear in that panel, instead of in panel CE. A comparison of the secondary systems of the two trusses is given later.

Bottom-Chord Stress eg—Referring to section 1-1, Fig. 1, it is seen that the algebraic sum of the horizontal components of stresses EG, d′e′ and eg must equal zero, which furnishes a simple method of determining stress eg. Thus, with the unit load in any position outside of panel CE, stress d′e′ is zero, and the horizontal components of stresses EG and eg must be equal in magnitude and opposite in direction. Therefore equations 1 and 2 apply for the horizontal component of stress eg, which, however, is tensile instead of compressive. The corresponding influence lines are ne and ac, Fig. 2 (b), drawn as explained for Fig. 2 (a).

STRESS FOR UNIT LOAD AT d′

With the unit load at d′, the horizontal component of stress d′e′ ($= p_1/h_1$) acts in a direction opposite to that of the horizontal component of stress EG, whose value is given by equation 3. Therefore

$$\text{horizontal component eg} = \frac{l - r + p_1}{l} \times \frac{r}{h} - \frac{p_1}{h_1} \qquad (4)$$

To complete the influence line, Fig. 2 (b), prolong ne to d′, and lay off d′d equal to p_1/h_1. Since the ordinate to d′ is equal to the first expression in the right-hand member of equation 4, the ordinate to d is equal to equation 4, and the complete influence line is acden.

The ordinates to this line must be multiplied by P''/P to obtain the value of stress eg.

The influence line for this stress is of different form from that for stress eg of the Pettit truss, Fig. 3 (a), which, since the latter member is unaffected by the secondary system, may be represented by acen, Fig. 2 (b). An ordinate between ce and cde therefore represents a *decrease* in the horizontal component of the chord stress eg induced by the secondary system, the value of d″d being equal to p_1h_2/h_1h; while it is evident that an ordinate between cde and cd′e at any point is equal to the horizontal component of stress d′e′ from a unit load at that point.

Thus it is seen that while in the Pettit truss the secondary system affects the stresses in only one chord system, in the K-truss both chord systems are affected, the stress in the top chord being increased and that in the bottom chord decreased. If such members as d′e′, Fig. 1, were arranged to act as struts instead of ties by being connected to the upper of the two diagonals, the effect would be to *decrease* the top and *increase* the bottom-chord stress. Finally, if the secondary systems in either truss are removed, the analysis reduces essentially to that of the simple Pratt truss.

WEB STRESSES

Diagonal Stress e′G—Imagine the truss to be divided into two parts by a vertical section 2-2 cutting four members, Fig. 1. To determine stress e′G, it is desirable to select, if possible, a moment center about which the algebraic sum of the moments of the stresses in the other three members cut will be zero, thus leaving stress e′G as the only unknown factor in the moment equation. O_1 is such a moment center for any position of the unit load outside of panel CE. This may be demonstrated as follows: As previously explained, for such a position of the load the horizontal components of stresses EG and eg are equal in magnitude and opposite in direction. Hence the resultant of these stresses must be a vertical force through their intersection O; this resultant and stress e′f′ both traverse O_1; therefore the sum of the moments of stresses EG, eg and e′f′ about O_1 is zero.

The stress in e′G for loads outside of panel CE may therefore be determined in exactly the same way as the stress in any diagonal web member of a Pratt truss with inclined chords, the only difference being that the moment center in the latter case lies at the intersection of the chords. Thus, for the unit load in any position from f′ to the right, letting x equal the distance from the right support, the compressive stress is

$$\text{e}'\text{G} = \frac{x}{l} \times \frac{v}{z} \qquad (5)$$

Similarly, for the unit load in any position from c′ to the left, letting x' equal the distance from the left support, the tensile stress is

$$\text{e}'\text{G} = \frac{x'}{l} \times \frac{l+v}{z} \qquad (6)$$

For the unit load at e′, stress d′e′ is still zero and equation 6 applies by substituting r for x'.

With the unit load at d′, consider that part of the truss to the right of section 2-2. Since d′e′ is now stressed, the horizontal components of stresses EG and eg are no longer equal, their algebraic sum now being equal to the horizontal component of stress d′e′ or p_1/h_1, as previously explained. The resultant of stresses EG and eg is therefore no longer a vertical force traversing O_1, but an *inclined* force through O whose horizontal component is p_1/h_1. The moment of this force about O_1, equal to its horizontal component multiplied by OO_1, must now be included in the equation of moments of forces to the right of section 2-2, which may be written:

$$\text{e}'\text{G} \times z - R_n(l+v) - \frac{p_1}{n_1} \times \frac{t+P}{P} h_1 = 0,$$

or, since $R_n = \dfrac{r - p_1}{l}$, the tensile stress is

$$\text{e}'\text{G} = \frac{r-p_1}{l} \times \frac{l+v}{z} + \frac{p_1}{r'} \times \frac{t+P}{z} \qquad (7)$$

The influence line may now be constructed. Equations 5 and 6 are plotted in Fig. 2 (c) as nf and ac respectively. For convenient construction, note the familiar property that the intercepts aa′ and nn′ are equal, respectively, to v/z and $l+v/z$, showing that nf and ac prolonged intersect on the vertical through O_1. Prolong ac to e, and draw ef; s marks the position of a load for zero stress in e′G. Finally, prolong ge to its intersection d with the vertical line (4); the ordinate d′d is equal to equation 7, the proof of this construction being given in the figure. The complete influence line is therefore acdefn.

If d′e′ were removed, the influence line for stress e′G would become acefn, which is typical of trusses with single web systems, and, incidentally, is the form of influence line for stress c′E in Fig. 1, since there is no member at c′ corresponding to d′e′. Therefore the ordinates from ce to cde, Fig. 2 (c), represent the increase in stress e′G induced by the secondary system in panel CE, the value of d″d being equal to the last term in equation 7. For a K-truss without sub-panels the influence line would take the familiar form acegn.

Diagonal Stress e′f′—The moment center for this stress is O_2, and the analysis is similar to that for stress e′G. Thus, equations 5 and 6 apply by substituting z_1 for z and reversing the character of stress. The segments ac and efn of the influence line, Fig. 2 (d), require no explanation beyond noting that the intercept aa′ is here equal to v/z_1.

With the unit load at d′, the analysis leading to equation 7 may be applied to stress e′f′, O_2 being the moment center instead of O_1. Then the value of the compressive stress is

$$\text{e}'\text{f}' = \frac{r-p_1}{l} \times \frac{l+v}{z_1} - \frac{p_1}{P} \times \frac{h_2}{h_1} \times \frac{t+P}{z_1} \qquad (8)$$

To complete the influence line, measure aa″ $= h_2v/h_1z_1$, prolong oa″ to g″ and draw g″e to its intersection d with line (4); the ordinate d′d is equal to equation 8 as proved in the figure; or the last term in equation 8 may be computed and plotted directly as d″d. The ordinates from ce to cde represent the *decrease* in stress e′f′ induced by the secondary system in panel CE.

Diagonal Stress f′g—Stresses e′f′ and f′g are evidently equal except for loading in panel EG, which brings a floorbeam concentration at f′. Therefore the influence line for stress e′f′, Fig. 2 (d), applies to stress f′g except across panel EG; and the influence line for the stress f′g may be completed by determining the value of that stress for a unit load at f′.

For that position of the load, the triangle f′g′g, Fig.

1, may be considered as a polygon for the forces at joint f′, the length g′g representing the unit load. The force represented by gf′ ($= i_1'/h_1'$) is the resultant of stresses e′f′ and f′g due to that load. Stress e′f′ is tensile—equal to the ordinate f′f, Fig. 2 (d)—and stress f′g is compressive or tensile according as stress e′f′ is less or greater than i_1'/h_1'. The former condition usually obtains, and the following equation gives the value and sign of the stress:

$$f'g = e'f' - \frac{i_1'}{h_1'} \qquad (9)$$

It will be convenient to express i_1' in terms of z_1. Thus $i_1' = pi_1/P$, and $i_1 = th'/z_1$ (from the value for z_1, Fig. 1); substituting, equation 9 becomes

$$f'g = e'f' - \frac{p}{P} \times \frac{h'}{h_1'} \times \frac{t}{z_1} \qquad (10)$$

Now, since f′f, Fig. 2 (d), is equal to stress e′f′, if a point f_1 is found such that $ff_1 = \frac{p}{P} \times \frac{h'}{h_1'} \times \frac{t}{z_1}$ $f'f_1$ will equal stress f′g. Measure $a'a''' = \frac{h'}{h_1'} \times \frac{v}{z_1}$ prolong oa‴ to e″ and draw e″g; f_1 is the point of intersection of e″g with the vertical line (6). The proof of this construction is given in the figure. By using equation 10 rather than equation 9 to locate f_1, it is clearly shown that ef_1g is always concave upward, since e″ cannot lie above e; further, that e″g is a locus for point f_1 for various values of p, assuming panel point g′ fixed in position—that is, h_1' constant. The complete influence line is $acdef_1gn$; s_1 marks the position of a load for zero stress in f′g.

The influence line for the corresponding member of a Pettit truss—f′g, Fig. 3 (a)—takes the form acegn, Fig. 2 (d); that is, it is a straight line across panel EG, as distinguished from the broken line ef_1g for the K-truss. It is interesting to note, however, that ef_1g approaches the straight line eg as a limit if the point g′ in the K-truss is assumed to approach G, since then the ratio h'/h_1' approaches unity and e″ approaches e, Fig. 2 (d). In fact, to carry such a change in the K-truss to the limit of coincidence of g′ with G, e′ with E, etc., virtually destroys its identity, and reduces it to the Pettit type.

Sub-Diagonal Stress f′g′—This stress is zero except for loading in panel EG, and for a unit load at f′ its value (tension) is

$$f'g' = \frac{i'}{h_1'} \qquad (11)$$

For such a simple case an influence line is unnecessary, the maximum stress being developed concurrently with the maximum floorbeam concentration at f′. An interesting point to observe, however, is that the vertical ordinate included between the lines ef_1g and ef_2g, Fig. 2 (d), at any point is equal to stress f′g′, due to a unit load at that point, multiplied by z_2/z_1, in which z_2 is the lever arm (not shown) of stress f′g′ about O_2. This may be proved from the construction of the figure, or by taking moments about O_2 of the stresses and external forces to either side of section 3-3, Fig. 1, with the unit load at f′. A similar relation exists with the Pettit truss, in which case the corresponding influence lines are eg and ef_2g, Fig. 2 (d), the member f′g′ becomes f′G, Fig. 3 (a), and the values z_2 and z_1 represent, respectively, the lever arms about the intersection of the chords of members f′G and f′g, Fig. 3 (a).

Stresses in Vertical Posts—These stresses follow directly from those in the diagonals. Thus, by resolving stress e′G into components at G in the directions EGK and Gg′, it is seen that

$$Gg' = e'G \times \frac{h_2}{i} \qquad (12)$$

and that the stresses in the two members are of opposite character. Therefore, Fig. 2 (c) may be used as the influence line for stress Gg′ by multiplying the ordinates by h_2/i. In like manner the ordinates to the influence line for stress f′g, multiplied by h_1/i_1, represent the stress in the lower part g″g of post g′g; the stress in the upper part g′g″ is equal to that in the lower except when there is a floorbeam load at g″.

The construction of certain of the influence lines is simplified if the sub-diagonal and main diagonal in a given panel are parallel. Thus, if d′e′ and c′E are parallel, line cd in Fig. 2 (b) will be parallel to en, and cd in Fig. 2 (d) will be parallel to eg; while if f′g′ is parallel to e′G, line ef_1 in Fig. 2 (d) will be parallel to fn. No convenient relations seem to obtain, however, for the usual case of equal length of sub-panels ($p = P/2$, etc.).

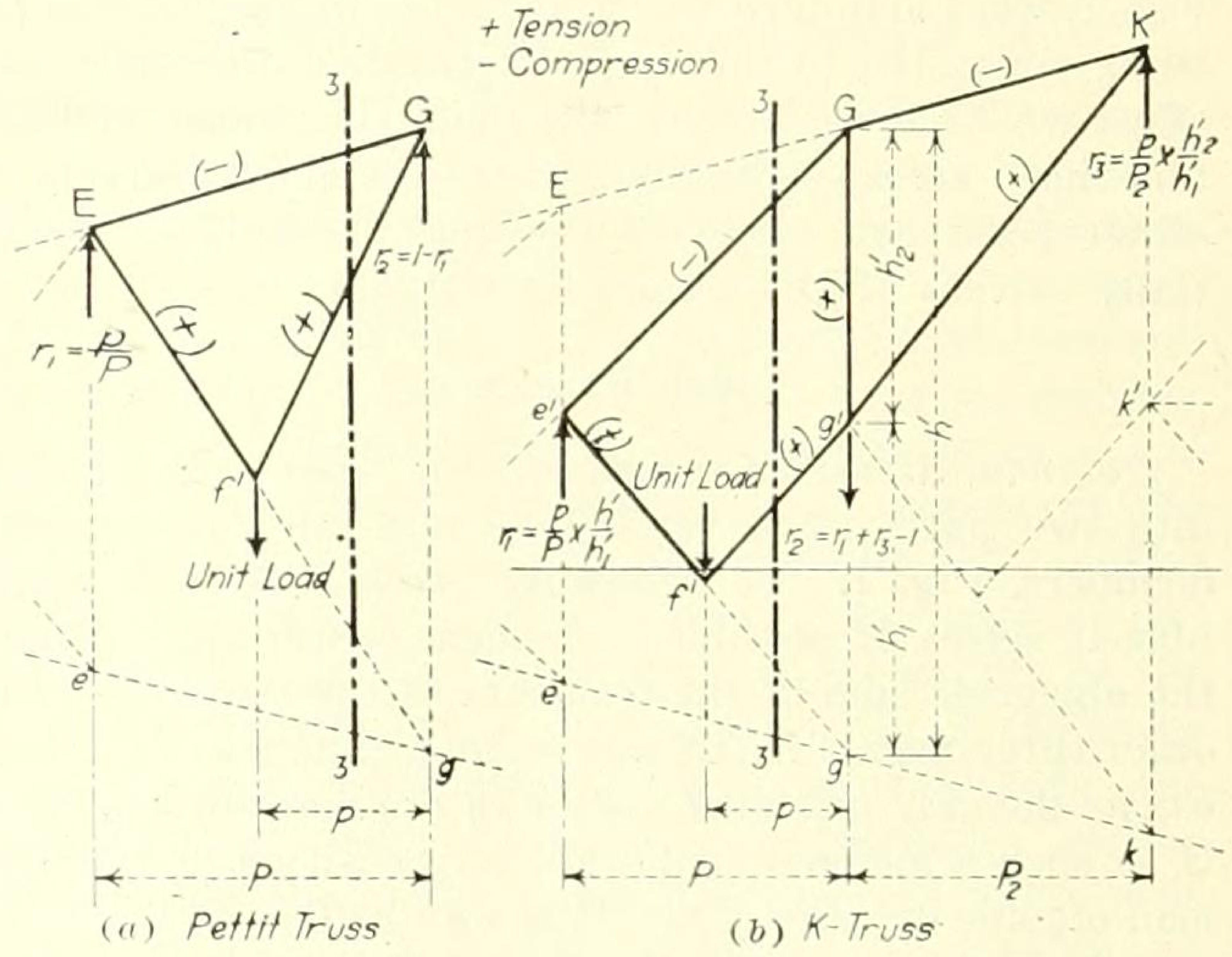

FIG. 3
Showing Action of Secondary Systems

COMPARISON OF "SECONDARY SYSTEMS"

A comparison between the so-called "secondary systems" of the Pettit and K-trusses can now be made. In the Pettit truss, Fig. 3 (a), the frame Ef′G acts as a secondary truss system in transmitting any load at f′ in proportionate parts to the main panel points E and G. In the K-truss, Fig. 3 (b), for a unit load at f′, the secondary truss system is e′f′g′KG (shown by full lines), since e′G and g′K—in the absence of a member e′g′ corresponding to EG of the Pettit truss—must develop the horizontal components of stresses e′f′ and f′g′ respectively.

Evidently, then, the load at f′ is not transmitted proportionately to e′ and g′; nor is it transmitted proportionately to e′ and K, since the secondary truss—owing to the presence of the quadrilateral frame e′f′g′G—would be unstable if supported simply at these two points. The truss would be stable, however, if supported at e′, g′ and K.

Moreover, the reactions r_1, r_2 and r_3 at these points are not statically indeterminate, as they would be if the quadrilateral frame were braced by the insertion of members e′g′ or f′G. In fact, the presence of this frame, free to deform without change in length of its sides, furnishes the third condition (the other two being $\Sigma V = 0$ and $\Sigma M = 0$) required to determine the reactions by statics. (A familiar application of this principle is seen in the omission of diagonal bracing in the panel over steel supporting towers at the main piers of a cantilever bridge.) Thus, the value of r_1 may be found by applying the laws of equilibrium to joints f′ and e′; with r_1 known, r_2 and r_3 are readily determined. The values of these reactions, and the character of the stress induced in each member of the secondary system, are indicated in Fig. 3 (b). (For the general proportions shown, r_2 is negative.) The load at f′ must therefore be assumed to be replaced by *three* component concentrations at e′, g′ and K, equal and opposite to the reactions r_1, r_2 and r_3; the function of the secondary system being completed, f′g′ is no longer needed and the primary K-system transmits these concentrations to the supports.

Criterion for Stability Satisfied

It is appropriate here to direct attention to the fact that the K-truss, in either its primary form or with sub-panels, satisfies the familiar criterion for stability expressed by the equation $m = 2j - 3$, in which m is the number of members and j is the number of joints; so that the presence of the quadrilaterals e′f′g′G, etc., in the truss with sub-panels does not necessarily imply instability. That is to say, a truss does not of necessity have to consist of an assemblage of triangles in order to be stable; and in this case such members as e′g′ or f′G, inserted to reduce the truss to an assemblage of triangles, would be redundant, thus making the stresses in general statically indeterminate, as shown in the preceding paragraph by independent consideration of the secondary truss.

This analysis of the secondary system may be used to advantage in computing stress f′g, in either the Pettit or K-truss, by algebraic methods, since the sub-diagonal is eliminated from the forces at section 3-3, Fig. 1, without stressing f′g, which is not in the secondary system. On the other hand, its use in the determination of chord stress EG by the elimination of d′e′ (see section 1-1, Fig. 1) complicates rather than simplifies the work, since the stress in EG induced by its action in the secondary system must be determined and included in the final equation for stress.

The secondary system k′mn′m′ in panel KN, Fig. 1, serves to transmit a load at m proportionately to k′ and n′, as in the Pettit truss, after which the secondary members may be considered removed and the stresses in the primary K-system determined by the methods which have been developed, noting, however, that k′m acts as a member of both the secondary and primary systems.

[TO BE CONCLUDED]

Frontier Humor in an Official Report

In the last annual report of the Board of Road Commissioners for Alaska it was noted that "Alaska abounds in mineral wealth, fish and road-commission critics."

Denver Advised to Purchase Its Present Waterworks

Investigations Show Cost of a New System to Be Double That of Present Fixed Valuation—Existing Supply Ample

THE cost of a proposed municipal waterworks system for Denver has been estimated at $27,479,500, a figure more than double that of $13,415,899 given in a special master's report as the valuation of the existing Denver Union Water Company's works. The wide variation is due to the necessity of obtaining a supply from the Blue River, on the western side of the Continental Divide, as the remaining local sources of supply were found to be insufficient. In a report recently submitted to the Public Utilities Commission of Denver by H. A. Kluegel, chief engineer and G. H. Wilhelm, consulting engineer for Shirley Houghton, successor to the Van Sant-Houghton Company, engineers and contractors, of Denver and San Francisco, it is also claimed that the present supply is sufficient to meet Denver's needs for the next 20 years, and a purchase of the present waterworks system at the stipulated valuation is recommended.

The Situation

The Public Utilities Commission, on July 27, 1915, employed the Van Sant-Houghton Company to complete the engineering work and to prepare plans, specifications and estimates for a complete and adequate system of waterworks to replace the existing system, owned by the Denver Union Water Company. A year previous to this, the city council had passed an ordinance regulating and fixing the charges by the Denver Union Water Company for water furnished the city and its inhabitants. The water company brought suit and received a favorable verdict in the United States District Court. As a result, the investigations of a new system were instigated and later the Van Sant-Houghton Company was employed.

An option agreement was executed Feb. 21, 1916, between the water company and the city granting to the latter an option to purchase the company's property at a valuation, as fixed by the special master's report, of $13,415,899, or as modified by certain additions and accrued depreciation. This agreement, in turn, bound the city to take an appeal of the rate case to the Supreme Court and to enter into no further contracts leading toward the duplication of the system. The city then instructed the Van Sant-Houghton Company to investigate also the condition of the existing waterworks system. Later, a supplemental contract was made calling for the discontinuance of all investigative work and the preparation of a preliminary report, the source of the following notes.

Investigating for a New Source of Supply

When the Van Sant-Houghton Company was employed, the Public Utilities Commission suggested that the water supply be developed from within the South Platte drainage area, which is adjacent to Denver, and presented for consideration the following: (1) The ownership and operation of certain existing ditches; (2) the acquisition and operation of the Antero reservoir and the high line canal; (3) the selection of a res-

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[IN PART I, *which appeared in last week's issue, page* 223, *Mr. Warwick presented the complete analysis for single-span conditions, and compared the K-truss secondary system with that used in the Pettit truss. In Part II the analysis is made for a cantilever arm, and methods for the determination of maximum stress in a typical web member are described both for simple span and for cantilever arm.*—EDITOR.]

Fig. 4 shows the truss acting as a cantilever arm; the free end is at a, and the effect of the anchorage may be represented for the purpose of this discussion by horizontal reactions at N and n. The application of the foregoing analysis to the determination of chord and web stresses in a cantilever arm will now be made.

Chord Stresses—Taking section 1-1, Fig. 4, it is evident that stresses EG and eg are zero for loads at and to the right of e'. With a unit load in any position from c' to the left, at a distance x from the free end, stress d'e' is zero, and taking moments about e or E, there is obtained,

$$\text{horizontal component EG} = \text{horizontal component eg} = \frac{r - x}{h} \qquad (13)$$

Stress EG is tensile and stress eg compressive, and each becomes maximum when the load is at a.

With the load at d', it has been seen that the algebraic sum of the moments about e of stress d'e' and the load is zero. Therefore stress EG is zero, which is also evident from other considerations. Then, from $\Sigma H = 0$ at section 1-1, the horizontal components of stresses eg and d'e' are equal, or

$$\text{horizontal component eg} = \frac{p_1}{h_1} \qquad (14)$$

The influence lines aa'cdn and aa'cden, Fig. 4 (a) and (b), may now be constructed, and require no further explanation. If the members c'E and d'e' are parallel, line cd in Fig. 4 (b) will be horizontal.

Web Stresses—Considering e'G, take section 2-2, Fig. 4. O_1 is the moment center. Loads in any position from f' to the right do not stress e'G. Placing a unit load in any position from c' to the left, stress d'e' is zero, and the tensile stress in e'G is

$$\text{e'G} = \frac{x + v}{z} \qquad (15)$$

Equation 15 applies for the unit load at e' by substituting r for x. For the unit load at d', the analysis leading to equation 7 may be applied; or, more simply, moments may be taken about g, the intersection of e'f' and eg, since for this condition of loading stress EG is zero. The lever arm of e'G about g being Ph'/i $(= Pz/t)$, the following equation for tensile stress is obtained:

$$\text{e'G} = \frac{P + p_1}{P} \times \frac{t}{z} \qquad (16)$$

The influence line for stress e'G is aa'cdefn, Fig. 4 (c); d is located by prolonging ge to the vertical through d', for since the ordinate e'e $= t/z$, d'd must equal equation 16.

The analysis for stress e'f' is similar to that for stress e'G, and leads to the influence aa'cdefn, Fig. 4 (d). The influence line for stress f'g is aa'cdef$_1$gn, Fig. 4 (d), the ordinate ff$_1$ being equal to stress f'g from a unit load at f' $(= i'_1/h_1')$. The method of constructing Fig. 4 (d) is similar to that used for Fig. 2 (d). If the members e'G and f'g' are parallel, line ef$_1$ will be horizontal.

The stresses in the verticals follow directly from those in the diagonals, as previously explained.

A complete analysis should include the effect of loads on the suspended span. This may readily be done by constructing the influence lines for such loads, which will consist of straight lines, shown in part by the lines s'a', Fig. 4, extending from the points a' to the intersection of the base lines na, prolonged, with a vertical through the far end of the suspended span.

DETERMINATION OF MAXIMUM STRESSES

The methods of stress analysis which have been described, although developed for a moving concentrated load of unit intensity, may obviously be applied to the determination of dead and live-load stresses by algebraic methods. For complicated truss systems, the dead-load stresses are often determined graphically by constructing a stress diagram. In the case of live-load stresses, however, which in general become maximum for different members under different conditions of loading, the use of stress diagrams is not so convenient; and the stresses are in general preferably determined either algebraically or from influence lines, or by a combination of these two methods.

Simple Span—As an illustration, let it be required to determine the maximum tension in e'G due to a concentrated load system, considering the truss to act as a simple span. Referring to the influence line, Fig. 2 (c), it is seen that the loads must extend over the portions a of the span, as shown in Fig. 5, with the heavier ones near d' and e'. The exact position of the load system for maximum stress may be found as follows:

Referring to Fig. 5, stress e'G $= M/z$, in which M is the moment about O_1 of stresses e'f', EG and eg at section 2-2 and the external forces to the right, namely, the reaction R_n and the floorbeam concentration at f' $(= r_f)$. The moment of stress e'f' about O_1 is zero. Referring to the analysis leading to equation 7, the moment about O_1 of the resultant of stresses EG and eg is equal to its horizontal component, which has the value $r_d p_1/h_1$, multiplied by OO_1. Therefore

$$M = R_n(l + v) - r_f u + r_d \frac{p_1}{h_1} \times \frac{t + P}{P} h_1 \qquad (17)$$

in which R_n, r_f and r_d have the values given in Fig. 5. To determine the condition or criterion for which M, and therefore stress e′G, will be maximum, equate dM/dx to zero:

$$\frac{dM}{dx} = \frac{W(l+v)}{l} - \frac{W_3 u}{p_3} + \frac{p_1}{p_2} \times \frac{t+P}{P}\left(W_1 - \frac{P_1}{p_1}W_2\right) = 0$$

Solving,

$$\frac{l+v}{l}W = \frac{u}{p_3}W_3 - \frac{p_1}{p_2} \times \frac{t+P}{P}\left(W_1 - \frac{P_1}{p_1}W_2\right) \qquad (18)$$

For maximum stress, therefore, the load system must occupy such a position that equation 18 is satisfied, which can occur only as some wheel (the critical wheel)

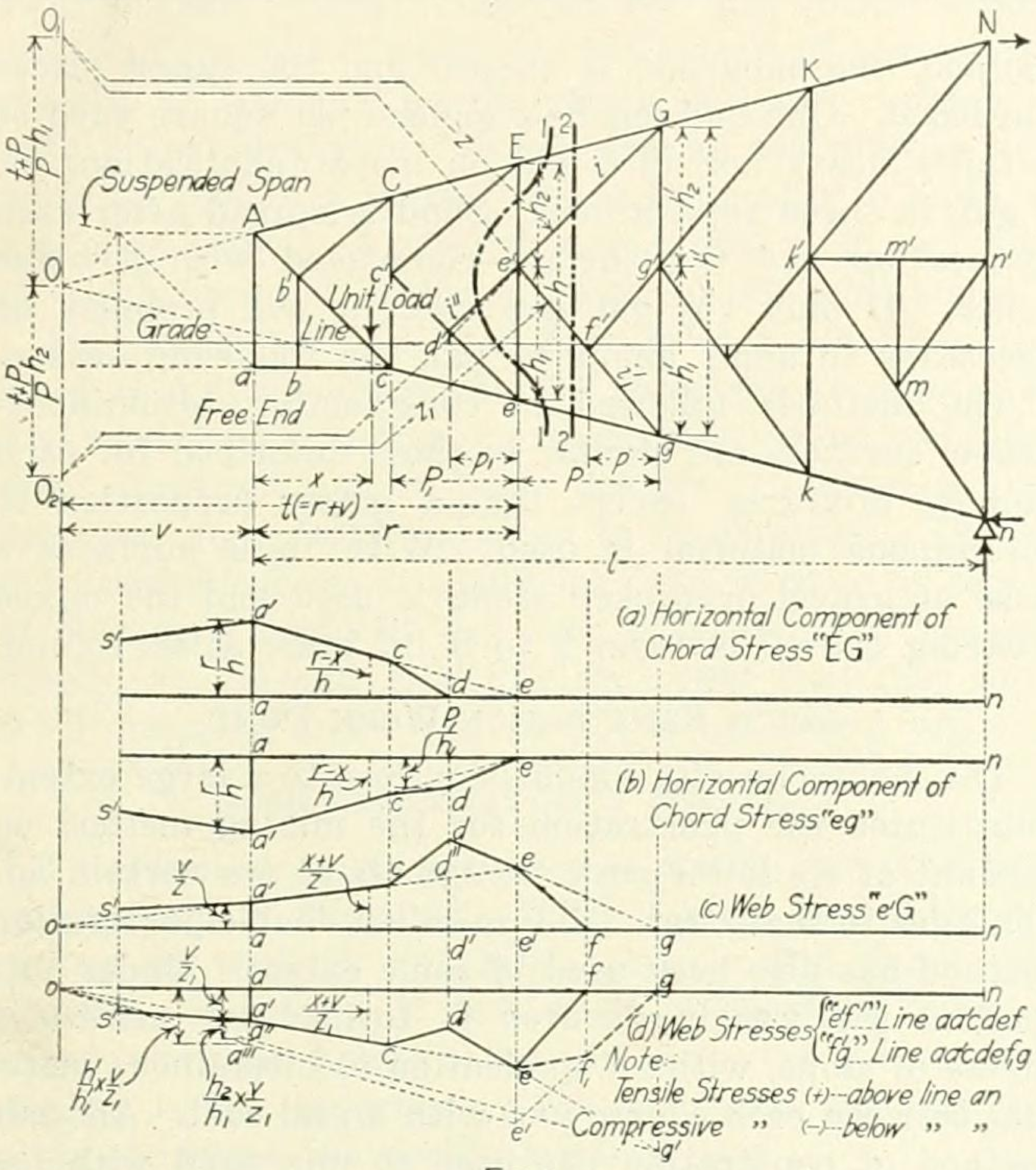

FIG. 4
Influence Lines for Stresses-Truss Acting as Cantilever Arm

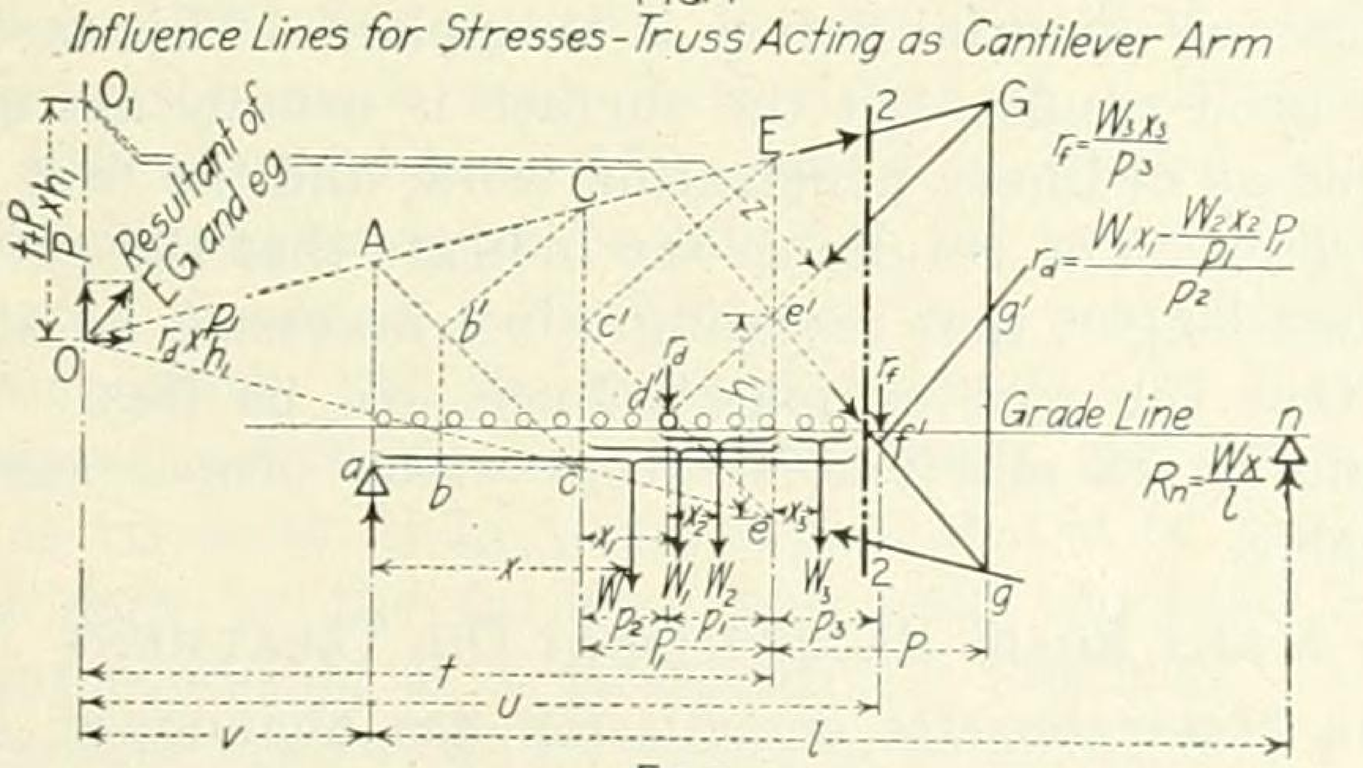

FIG. 5
Position of Concentrated Load System for Maximum Tension in e′G (Simple Span)

passes d′ or e′. In Fig. 5, the critical wheel is shown at d′, which is the more favorable position for satisfying the criterion, since the influence line at that point, Fig. 2 (c), is more sharply concave downward than at e′. With the load system in position, M may be computed from equation 17; whence stress e′G = M/z.

Or, the stress may be found by summing the products of each load by the length of the ordinate to the influence line beneath the load, such a summation being facilitated by using a rider on the scale to add the ordinates under loads of equal intensity. This method is especially convenient when the algebraic expression for stress and the criterion for maximum are complicated, as, for example, in determining the maximum compressive stress in f′g, for which the influence line, Fig. 2 (d), is relatively complicated. In that event, the position of load may be approximated with sufficient accuracy by trial.

If an equivalent uniform load is used in place of the concentrated load system, as is frequently done in long-span bridges, the conditions are simplified exactly as in the case of the usual truss analysis.

Anchor Arm—The analysis for maximum stresses in a simple span applies to an anchor arm under its own loading. Loads on the cantilever arm and suspended span cause an uplift at the anchorage, and the negative reaction developed there may be treated as a load at the end of the anchor arm acting as a cantilever arm. The stresses in the anchor arm from this condition of loading may most readily be determined by constructing a stress diagram.

Cantilever Arm—The analysis for maximum stresses in a cantilever arm is slightly different from that for a simple span. Calling the length of the suspended span l_s, the total load in that length W_s, and the distance of the center of gravity of W_s from the far support x_s, the proportion of W_s transmitted to a, Fig. 4, is $W_s x_s / l_s$. Then for maximum stress in e′G, for example, the following equation of moments about O_1 of stresses and external forces to the left of section 2-2 may be written, using the notation of Fig. 5:

$$M = \text{e}'\text{G} \times z = \frac{W_s x_s}{l_s} v + W(x+v) - r_f u - r_d \frac{p_1}{h_1} \times \frac{t+P}{P} h_1 \qquad (19)$$

Substituting the values of r_f and r_d, and equating dM/dx to zero, the criterion for maximum stress is

$$\frac{v}{l_s}W_s + W = \frac{u}{p_3}W_3 + \frac{p_1}{p_2} \times \frac{t+P}{P}\left(W_1 - \frac{P_1}{p_1}W_2\right) \qquad (20)$$

With the loading in position for maximum, the stress may be found by either of the methods outlined for simple spans.

35-Mile Road in Oklahoma Being Built by Convict Labor—Men Are Not Paid

Pollatalomie County, Oklahoma, the state and the U. S. Office of Public Roads and rural Engineering are co-operating in building a 35-mile road which passes through the important towns of that county. Necessary funds and a portion of the equipment are furnished by the county, the state providing the remainder of the equipment, 50 negro prisoners and four employees. The work is under the supervision of a U. S. road engineer. The National Committee on Prisons is watching the experiment with interest as a step toward affording opportunity for work to many Oklahoma convicts. The framers of the Oklahoma constitution took an advanced attitude when they inserted a provision in the constitution prohibiting the contracting of convict labor. The state has, therefore, no contract system to abolish and is in a position to forge ahead in building up its prison industries. The work is undertaken as a demonstration of what can be done with state convict labor. As no wages are paid to the convicts, cost figures are not easily available. Other states have found it practicable to pay the prisoners for their work with resulting greater efficiency.